# Technological University Dublin 

First Year Engineering Entrance Examination 2019

In<br>MATHEMATICS

Easter 2019

Attempt ALL 5 QUESTIONS
Time Allowed: 3 hours

All questions carry equal marks

Maths Tables and graph paper are available for use

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1. (a) Express $z=(6-2 i)(4-7 i)$ in polar form and calculate $z^{2}$. Express the results both in polar and rectangular forms.
(7 Marks)
(b) Find $a$ if $z=2+i$ is a root of $2 z^{2}+3 z+2 a-14+3 i=0$. (7Marks)
(c) Simplify the following expression involving indices:

$$
\frac{x^{5} y^{2} x^{3}+x^{4} y^{5}-y^{5} x^{7} y^{4}}{x^{4} y^{3}}
$$

(6 Marks)
2. (a) An archer hit the target with probability 0.8 . If she takes 10 shots find the probability that she misses 2 or more (10 Marks)
(b) The mean height of a group of 200 people is 164 cm with a standard deviation of 6 cm . Assuming the heights are normally distributed find the probability of a person's height being:
(i) less than 164 cm
(ii) more than 166 cm
(iii) greater than 176 cm
(10 Marks)
3. (a) Make $x$ the subject of the formula: $y+2 x b=\frac{x}{2 b}+x 9 b$
(6 Marks)
(b) Solve for $x$ :
(i) $\log _{10}(x+1)+\log _{10}(x-1)=3$
(ii) $\ln \left(\frac{x-2}{x-3}\right)=2$
(6 Marks)
(c) In a chemical reaction, the amount of material in grams after $t$ hours is given by

$$
M=31 e^{0.3 t}
$$

(i) What is the initial amount of $M$ ?
(ii) How much material is present after 10 hours and estimate how long it will take for $M$ to reach 100 grams.
(8 Marks)
4. (a) Given the following :

$$
\begin{aligned}
& C_{1} \text { is the circle } x^{2}+y^{2}+2 x-2 y-23=0 \\
& C_{2} \text { is the circle } x^{2}+y^{2}-14 x-2 y+41=0
\end{aligned}
$$

Prove that both circles touch externally and find the point of contact. (6 Marks)
(b) Find the equation of the line that passes through the point of intersection of the lines $3 x+2 y-1=0$ and $2 x-y+7=0$ and is perpendicular to the line $4 y+4 x=7$. (6 Marks)
(c) A building site is in the form of a quadrilateral as shown below. Determine the length of the perimeter of the site.
(8 Marks)

5. (a) Find values of the first derivatives of the following at the given points:
(i) $f(x)=\left(x^{2}+7 x-4\right)^{2}$ at $x=3$
(ii) $g(x)=\left(4 x^{2}-11 x\right)\left(11 e^{2 x}\right)$ at $x=0$
(6 Marks)
(b) Given the function $y=2 x^{3}-7 x^{2}-14 x+11$. Find the two turning points and specify if they are maximum or minimum points.
(6 Marks)
(c) Find the area under the curve $y=117+x$ between the values $x=2$ and $x=11$ (8 Marks)

